Sem IV

Unit :- 01.

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Electrodynamics & Electromagnetism (T)

Topic :-

(1) Maxwell's first equations

$\nabla \cdot E = \frac{\rho}{\epsilon_0}$ may be written as

$$\int_V (\nabla \cdot E)\, dV = \int_V \left(\frac{\rho}{\epsilon_0}\right) dV \qquad \text{--- (14)}$$

where V is any arbitrary volume. Applying Gauss's divergence theorem

$$\oint F \cdot ds = \int_V \operatorname{div} F \, dV$$

we get

$$\oint E \cdot ds = \frac{1}{\epsilon_0} \int_V \rho \, dV$$

i.e the outward flux of E throughout any closed surface S is $\left(\frac{1}{\epsilon_0}\right)$ times the net charge inside the volume enclosed in S.

(2) Equation (2) may be written as

$$\int_V (\nabla \cdot B)\, dV = 0,$$ where V is any arbitrary volume.

on applying Gauss's divergence theorem, we get

$$\oint B \cdot ds = 0 \quad (17)$$

i.e the outward flux of magnetic induction vector B throughout any closed surface S is zero.

$$\oint (\nabla \times E) \cdot ds = -\oint \left(\frac{\partial B}{\partial t}\right) \cdot ds$$

where S is any surface bounded by a curve C.

Applying stoke's theorem

$$\oint_C F\,dl = \oint_S (\text{curl}\,F)\,ds$$

we get $\oint_C E \cdot dl = -\oint_S \frac{\partial B}{\partial t} \cdot ds = -\frac{\partial}{\partial t}\oint B \cdot ds$ (18)

where S is any surface bounded by a curve C.

This equation may be started as: the induced e.m.f around curve C is equal to minus the rate of change of magnetic flux through the surface enclosed by the curve C.

(4) Maxwell's forth equation become

$$\oint_S B \cdot dl = \mu_0 \int \left(J + \epsilon_0 \frac{\partial E}{\partial t}\right) \cdot ds \quad (19)$$

i.e. the magnetomotive force around C is equal to the sum of the free current density and the displacement current density linking C.

The first two eqn's show that the net flux of lines of E or B is zero throughout any arbitrary volume of no source. They show that lines of E and B are continuous. The third equation shows that the electric field due to the variation of B is not only a statement of Faraday's law, but a property of space independent of the presence of a conducting loop of wire. The fourth eqn's shows the new concept of magnetic field generation by displacement current.

potential Formulations of Electrodynamics

$$B = \text{curl}\, A \quad \text{—— (20)}$$

$$\text{curl}\left(E + \frac{\partial A}{\partial t}\right) = 0 \quad \text{—— (21)}$$

$$E + \frac{\partial A}{\partial t} = \nabla \phi$$

$$E = -\nabla \phi - \frac{\partial A}{\partial t} \quad \text{—— (22)}$$

$$\nabla^2\phi + \frac{\partial}{\partial t}(\nabla \cdot A) = -\frac{\rho}{\epsilon_0} \quad \text{--- (23)}$$

$$\nabla \times (\nabla \times A) = \mu_0 J - \mu_0\epsilon_0 \nabla \frac{\partial \phi}{\partial t} - \mu_0\epsilon_0 \frac{\partial^2 A}{\partial t^2} \quad \text{--- (24)}$$

Using the vector identity

$$\nabla \times (\nabla \times A) = \nabla(\nabla \cdot A) - \nabla^2 A$$

and rearranging the terms.

$$\left(\nabla^2 A - \mu_0\epsilon_0 \frac{\partial^2 A}{\partial t^2}\right) - \nabla\left(\nabla \cdot A + \mu_0\epsilon_0 \frac{\partial \phi}{\partial t}\right) = -\mu_0 J \quad \text{--- (25)}$$

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